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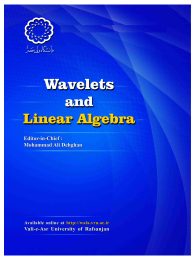

Amendment to the result on the trace of Haar wavelets matrix

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Abstract

Trace and some other interesting properties of Haar wavelets matrix of size 2^K are studied by Shiralashetti and Kumbinarasaiah [6], results related to the trace of Haar wavelets matrix derived based on K is even or odd and the same is concluded in Theorem 3.2. This article deals with the trace of Haar wavelets matrix in depth by identifying and overcoming the pitfalls occur in the proof of Theorem 3.2.

1. Introduction and Preliminaries

Haar wavelet is the only wavelet having analytic expression with compact support which were introduced by Alfred Haar [2] and treated as basis for other family of wavelets. Analytic

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expression and compact support allow us to integrate Haar wavelets arbitrary number of times and local analysis of functions respectively in easier way. Haar wavelets are applied in almost all branches of science and engineering [1, 3, 4, 5]. Discretization of Haar wavelet transform leads to a matrix called Haar wavelets matrix or Haar matrix. Some of the results on Haar wavelets matrix are obtained using linear transformation [6]. One of the quite attractive results on Haar matrix is its trace. In this article, a counterexample is provided to show that the conclusion of Theorem 3.2 [6] does not holds and proposed an algorithm to compute the trace of Haar wavelets matrix.

The rest of the paper is organized as follows: Section 2 contains the necessary basic concepts related to Haar family and the trace of Haar wavelets matrix existing in literature. Section 3 contais a counterexample to illustrate the validation of conclusion on the trace of Haar matrix discussed in section 2. Section 4 deals with occurance of pitfalls in the proof of existing results on the trace of Haar matrix. Section 5 contains an amendment to result on trace of Haar matrix mentioned in section 2 and an algorithm to compute the same.

2. Haar family and the trace of Haar wavelets matrix existing in literature

2.1. *Haar family*

For a given resolution $J \in \mathbb{N}$, we have 2^{J+1} Haar wavelets on $[0,1)$, among these wavelets two wavelets are important namely father and mother wavelets, remaining $2^{J+1}-2$ are called daughter wavelets and these can be obtained through scaling(dilation) and translation(shift) of mother wavelet. This family is an orthogonal subset of the Hilbert space $L_2([0,1))$ with the inner product $\langle f,g\rangle = \int_0^1 f(t)\overline{g(t)}dt$. For the sake of completeness, we recall the Haar family and its properties defined on continuous domain $[0,1)$[1, 4]:
Father wavelet:

$$h_1(t) = \begin{cases} 1, & t \in [0,1) \\ 0, & otherwise. \end{cases} \tag{2.1}$$

Mother wavelet:

$$h_2(t) = \begin{cases} 1, & t \in [0,0.5) \\ -1, & t \in [0.5,1) \\ 0, & otherwise. \end{cases} \tag{2.2}$$

Daughter wavelets: For $j \in \{1,2,\ldots,J\}$, there are 2^j daughter wavelets and are given by

$$h_i(t) = h_2(2^j t - k) = \begin{cases} 1, & t \in [\frac{k}{2^j}, \frac{k+0.5}{2^j}) \\ -1, & t \in [\frac{k+0.5}{2^j}, \frac{k+1}{2^j}) \\ 0, & otherwise, \end{cases} \tag{2.3}$$

where $i = 2^j + k + 1$ and $k = 0,1,\ldots,2^j-1$. Here j and k are called coarser resolution level and translation parameters respectively.

Definition (Compact Support): Let $f : \mathbb{R} \to \mathbb{R}$ be the function.Then the compact support of f is denoted as $Comp(f)$ and is defined by the smallest compact subset of $\mathbb{R}$ on which f is nonzero.

Example 2.1. $Comp(h_3) = [0, 0.5]$ and $Comp(h_4) = [0.5, 1]$.

Theorem 2.2. *For a fixed $j \in \{1, 2, \ldots, J\}$. The set of all j^{th} stage daughter wavelets $\{h_{2^j+1}, h_{2^j+2}, \ldots, h_{2^{j+1}}\}$ is orthogonal.*

Proof. Proof follows from the fact that $Comp(h_i)$ and $Comp(h_{i'})$ is either disjoint or singleton. □

Corollary 2.3. *For a given resolution $J \in \mathbb{N}$. The Haar family $\{h_i : i = 1, 2, ..., 2^{J+1}\}$ is an orthogonal set.*

Proof. Proof follows from the fact that area of each wavelet except the father wavelet is zero. □

2.2. *Haar wavelets matrix*

For a given $K = J + 1 (J \in \mathbb{N})$. Divide the interval $[0, 1]$ into 2^K subintervals and denote the l^{th} subinterval by $I_l = [\frac{(l-1)}{2^K}, \frac{l}{2^K}]$ for $l = 1, 2, \ldots, 2^K$. Midpoint of each subinterval is called the l^{th} collocation point and defined by $\frac{2l-1}{2^{K+1}}$. We denote the set of all collocation points by C and is defined by $C = \{t_l = \frac{2l-1}{2^{K+1}} : l = 1, 2, \ldots, 2^K\}$.

Haar wavelets matrix(H) of size $2^{J+1} = 2^K$ can be obtained by evaluating above 2^K Haar wavelets defined on $[0, 1)$ at the points of C and put them in rows of H. In other words $(i, l)^{th}$ entry of H is formulated as below:

$$H(i, l) = h_i(t_l), \quad \text{for} \quad i, l \in \{1, 2, \ldots, 2^K\}. \tag{2.4}$$

Behavior of each wavelet h_i for $i = 1, 2, \ldots, 2^K$ on C studied with the help of equations (2.1) – (2.3) and simplified versions are the following.

$$h_1(t_l) = \begin{cases} 1, & t_l \in [0, 1) \\ 0, & otherwise. \end{cases} \tag{2.5}$$

$$h_i(t_l) = \begin{cases} \pm 1, & t_l \in [\frac{k}{2^j}, \frac{k+1}{2^j}) \\ 0, & otherwise. \end{cases} \tag{2.6}$$

Let $C_i = \{t_l \in C : h_i(t_l) = \pm 1\}$. Arrange elements in C_i in increasing order and divide C_i into two subsets C_i^+ and C_i^-, $t_l \in C_i^+$ if and only if $h_i(t_l) = +1$ and $t_l \in C_i^-$ if and only if $h_i(t_l) = -1$. More precisely the various C_i(s) are:

$$\begin{cases} C_1 = C; C_1^+ = C; C_1^- = \emptyset. \\ C_i = \{t_{k\Delta_j+1}, \ldots, t_{(k+1)\Delta_j}\} \\ C_i^+ = \{t_{k\Delta_j+1}, \ldots, t_{k\Delta_j+\frac{\Delta_j}{2}}\} \\ C_i^- = \{t_{k\Delta_j+\frac{\Delta_j}{2}+1}, \ldots, t_{(k+1)\Delta_j}\} \end{cases} \tag{2.7}$$

Where $j \in \{0, 1, ..., K-1\}$, $\Delta_j = 2^{K-j}$, $i = 2^j + k + 1$, $k = 0, 1, 2, \ldots, 2^j - 1$.

Further the equations (2.5) and (2.6) simplified as below.

$$H(1, l) = 1, \quad \text{for} \quad l = 1, 2, ..., 2^K. \tag{2.8}$$

$$H(i, l) = \begin{cases} 1, & t_l \in C_i^+ \\ -1, & t_l \in C_i^- \\ 0, & t_l \in C - C_i. \end{cases} \tag{2.9}$$

Equations (2.8) and (2.9) describe each entry in the Haar wavelets matrix of size 2^K.

2.3. *The trace of Haar wavelets matrix existing in literature*

Main goal of this article is to compute the trace of Haar wavelets matrix of size 2^K for a given K. i.e., we have to compute $\sum_{i=1}^{2^K} H(i, i)$. Existing results related to this goal is the following.

Statement(Theorem 3.2 of Siddu Channabasappa Shiralashetti and Kumbinarasaiah [6]).
Let H be the $2^K \times 2^K$ Haar matrix. Then

$$Trace(H) = \text{tr}(H) = \begin{cases} 0 & \text{if} \quad K \quad \text{is odd} \\ K - 1 & \text{if} \quad K \quad \text{is even.} \end{cases} \tag{2.10}$$

3. Counterexample

We use equations:(2.7)-(2.9) to construct a counterexample to support that the conclusion of above statement [Equation (2.10)] is not valid in two parts.

Part-1 (K is odd): For $K = 7$ (Equation (2.10) is true for $K = 3$ and 5), H is a size of $2^7 = 128$. We show that $\text{tr}(H) \neq 0$.

Solution: Here $K = 7$, the set of all collocation points: $C = \{t_l = \frac{2l-1}{256} : l = 1, 2, ..., 128\}$. We construct the Haar matrix of size 128 row-wise by considering eight($K + 1$) cases.

Case 1 **[First row of H]**: For $i = 1$, we have $C_1 = C$; $C_1^+ = C$; $C_1^- = \emptyset$. The father wavelet h_1 is +1 at all collocation points. Hence all columns of 1^{st} row are 1. i.e., $H(1, l) = 1, \forall l$. Particularly

$$H(1, 1) = 1. \tag{3.1}$$

Case 2 **[Second row($j = 0$) of H]**: For $j = 0$ and $k = 0$, we obtain $i = 2^j + k + 1 = 2$. We have $C_2 = C$; $C_2^+ = \{t_l : l = 1, 2, ..., 64\}$; $C_2^- = \{t_l : l = 65, 66, ..., 128\}$. The mother wavelet h_2 is +1 at first half of collocation points and h_2 is -1 at second half of collocation points. Particularly

$$H(2, 2) = 1. \tag{3.2}$$

Case 3 **[Third and fourth rows($j = 1$) of H]**: For $j = 1$, we have 2^j daughter wavelets. Third($k = 0$) and fourth($k = 1$) rows of H can be obtained as follows.

For $i = 2^1 + 0 + 1 = 3$. We have $C_3 = \{t_l : l = 1, 2, ..., 64\}$; $C_3^+ = \{t_l : l = 1, 2, ..., 32\}$; $C_3^- = \{t_l : l = 33, 34, ..., 64\}$.i.e., nonzero entries in 3^{rd} of H are formulated as below.

$$H(3, l) = 1 \quad \text{for} \quad l = 1, 2, ..., 32; H(3, l) = -1 \quad \text{for} \quad l = 33, 34, ..., 64.$$

Particularly:

$$H(3, 3) = 1. \tag{3.3}$$

For $i = 2^1 + 1 + 1 = 4$. We have $C_4 = \{t_l : l = 65, 66, ..., 128\}$; $C_4^+ = \{t_l : l = 65, 66, ..., 96\}$;$C_4^- = \{t_l : l = 97, 98, ..., 128\}$.i.e., nonzero entries in 4^{th} of H are formulated as below.

$$H(4, l) = 1 \quad \text{for} \quad l = 65, 66, ..., 96; H(4, l) = -1 \quad \text{for} \quad l = 97, 98, ..., 128.$$

$$H(4, 4) = 0. \tag{3.4}$$

Case 4 [**Rows 5 to 8** ($j = 2$) **of** $\boldsymbol{H}$]: Here $i = 2^2 + k + 1$, for $k = 0, 1, 2, 3$. We obtain entries in 5-8 rows of H.
$k = 0$ implies $i = 5$. Hence $C_5 = \{t_l : l = 1, \ldots, 32\}, C_5^+ = \{t_l : l = 1, \ldots, 16\}, C_5^- = \{t_l : l = 17, \ldots, 32\}$. Nonzero entries in 5^{th} row of H formulated as below:

$$H(5, l) = 1 \quad \text{for} \quad l = 1, 2, \ldots, 16.; H(5, l) = -1 \quad \text{for} \quad l = 17, 18, \ldots, 32.$$

$k = 1$ implies $i = 6$. $C_6 = \{t_l : l = 33, \ldots, 64\}, C_6^+ = \{t_l : l = 33, \ldots, 48\}, C_6^- = \{t_l : l = 49, \ldots, 64\}$. In other words, nonzero entries in 6^{th} row of H formulated as below:

$$H(6, l) = 1 \quad \text{for} \quad l = 33, \ldots, 48.; H(5, l) = -1 \quad \text{for} \quad l = 49, \ldots, 64.$$

$k = 2$ implies $i = 7$. $C_7 = \{t_l : l = 65, \ldots, 96\}, C_7^+ = \{t_l : l = 65, \ldots, 80\}, C_6^- = \{t_l : l = 81, , \ldots, 96\}$. In other words, nonzero entries in 7^{th} row of H formulated as below:

$$H(7, l) = 1 \quad \text{for} \quad l = 65, \ldots, 80.; H(7, l) = -1 \quad \text{for} \quad l = 81, \ldots, 96.$$

$k = 3$ implies $i = 8$. $C_8 = \{t_l : l = 97, \ldots, 128\}, C_8^+ = \{t_l : l = 97, \ldots, 112\}, C_8^- = \{t_l : l = 113, , \ldots, 128\}$. In other words, nonzero entries in 8^{th} row of H formulated as below:

$$H(8, l) = 1 \quad \text{for} \quad l = 97, \ldots, 112.; H(8, l) = -1 \quad \text{for} \quad l = 113, \ldots, 128.$$

Therefore ,

$$H(5, 5) = 1, H(6, 6) = 0, H(7, 7) = 0, H(8, 8) = 0 \tag{3.5}$$

Case 5 [**Rows 9 to 16** ($j = 3$) **of** $\boldsymbol{H}$]: Nonzero entries in 9 to 16 rows of H are as follows:

$$H(9, l) = 1, \quad l = 1, 2, \ldots, 8; H(9, l) = -1, \quad l = 9, \ldots, 16.$$

$$H(10, l) = 1, \quad l = 17, 18, \ldots, 24; H(10, l) = -1, \quad l = 25, \ldots, 32.$$

$$H(11, l) = 1, \quad l = 33, 34, \ldots, 40; H(11, l) = -1, \quad l = 41, \ldots, 48.$$

$$H(12, l) = 1, \quad l = 49, 50, \ldots, 56; H(12, l) = -1, \quad l = 57, \ldots, 64.$$

$$H(13,l) = 1, \quad l = 65, 66, \ldots, 72; H(13,l) = -1, \quad l = 73, \ldots, 80.$$

$$H(14,l) = 1, \quad l = 81, \ldots, 88; H(14,l) = -1, \quad l = 89, \ldots, 96.$$

$$H(15,l) = 1, \quad l = 97, \ldots, 104; H(15,l) = -1, \quad l = 105, \ldots, 112.$$

$$H(16,l) = 1, \quad l = 113, \ldots, 120; H(16,l) = -1, \quad l = 121, \ldots, 128.$$

Therefore,

$$\begin{cases} H(9,9) = -1; H(10,10) = 0; H(11,11) = 0; H(12,12) = 0; \\ H(13,13) = 0; H(14,14) = 0; H(15,15) = 0; H(16,16) = 0. \end{cases} \tag{3.6}$$

Case 6 [**Rows 17 to 32** ($j = 4$) **of** $\boldsymbol{H}$]:Nonzero entries in the rows from 17 to 32 of H are listed below.

$$H(17,1) = 1, \quad l = 1, \ldots, 4; H(17,1) = -1, \quad l = 5, \ldots, 8.$$

$$H(18,1) = 1, \quad l = 9, \ldots, 12; H(18,1) = -1, \quad l = 13, \ldots, 16.$$

Nonzero entries in 18^{th} row obtained by shifting the nonzero entries in previous(17^{th}) row by 8 units (columns), use this procedure to obtain the remaining rows up to 32^{nd} row.

$$H(19,l) = 1, \quad l = 17, \ldots, 20; H(19,l) = -1, \quad l = 21, \ldots, 24.$$

$$\vdots$$

$$H(32,l) = 1, \quad l = 121, \ldots, 124; H(32,l) = -1, \quad l = 125, \ldots, 128.$$

Nonzero value lies on the diagonal of H is $H(19,19) = 1$. i.e.,

$$\begin{cases} H(i,i) = 0 \quad \text{for} \quad i \in \{17, 18, \ldots, 32\} - \{19\} \quad \text{and} \\ H(19,19) = 1. \end{cases} \tag{3.7}$$

Case 7 [**Rows 33 to 64** ($j = 5$) **of** $\boldsymbol{H}$]: Nonzero entries in 33^{rd} row are : $H(33,1) = 1, \text{for} \quad l = 1, 2; H(33,1) = -1, \text{for} \quad l = 3, 4$. To obtain nonzero entries in rows from: 34 to 64, we shift the nonzero entries in previous row by 4 columns. Hence nonzero value lies on the diagonal of H is $H(43,43) = -1$.

$$i.e., \; H(i,i) = 0 \quad \text{for} \quad i \in \{33, 34, \ldots, 64\} - \{43\} \quad \text{and} \quad H(43,43) = -1. \tag{3.8}$$

Case 8 [**Rows 65 to 128** ($j = 6$) **of** $\boldsymbol{H}$]: Nonzero entries in 65^{th} row are:$H(65,1) = 1$; $H(65,2) = -1$. To obtain nonzero entries in rows: 66 to 128, we shift the nonzero entries in previous row by 2 columns. Hence nonzero value contribution in the sum involving of trace of H is $H(128,128) = -1$. i.e.,

$$H(i,i) = 0 \quad \text{for} \quad i \in \{65, 66, \ldots, 128\} - \{128\} \quad \text{and} \quad H(128,128) = -1. \tag{3.9}$$

Use above equations (3.1)-(3.9) to compute the required tr(H).

$$\text{tr}(H) = \sum_{i=1}^{128} H(i,i),$$

$$\text{tr}(H) = \begin{cases} H(1,1) + H(2,2) + H(3,3) + H(5,5) + H(9,9) + \\ H(19,19) + H(43,43) + H(128,128), \end{cases}.$$

$$= 1 + 1 + 1 + 1 + (-1) + 1 + (-1) + (-1).$$

Therefore, tr(*H*) = 2(≠ 0).

Remark 3.1. If H is Haar wavelets matrix of size 2^K, then there are only $K+1$ nonzero values (± 1) among the sum involving of 2^K terms in the computation of trace of H.

Part-2(K is even) : Haar wavelets matrix of size $256 = 2^8$ i.e., $K = 8$ (Statement (2.10) is true for $K = 2, 4$ and 6). We show that $\text{tr}(H) \neq 7$.

Solution: Collocation points: $C = \{t_l = \frac{2l-1}{512} : l = 1, 2, \ldots, 256\}$.
Following are the $K + 1 = 8 + 1 = 9$ nonzero values in the computation of trace of H of size 256.

$$\text{tr}(H) = \begin{cases} H(1,1) + H(2,2) + H(3,3) + H(5,5) + H(9,9) + H(18,18) \\ +H(37,37) + H(86,86) + H(256,256). \end{cases}.$$

$$\text{tr(H)} = 1 + 1 + 1 + 1 + 1 + 1 + (-1) + 1 + (-1). \tag{3.10}$$

Therefore, tr(*H*) = 5(≠ 8 − 1).

4. Identification of pitfalls

Now, we look at the proof provided to the statement (2.10) [Theorem 3.2 [6]]. Proof contains two parts depends on K is even(first part) and odd(second part). Second part uses the first part. Hence it is enough to identify the pitfall in the first part. As there are $K+1$ cases and hence $K+1$ non zero values lie on the diaognal of H. Part-1 concluded that: only one nonzero is -1(Case: $K+1$), remaining are $+1$(Cases:1 to K), hence $\text{tr}(H) = K - 1$. In counterexample for $K = 8$, we found that there are two -1 values on the diagonal of H (Equation (3.10)).

5. Results and algorithm

To meet the goal of this article we have to concentrate on how to find the $K+1$ nonzero values lie on the diagonal of H of size 2^K. The journey to reach this goal is through the following theorem and algorithm.

Theorem 5.1. *Let H be a Haar wavelets matrix of size $n = 2^K (K > 1)$. Then*

$$\text{tr}(H) = 2 + \sum_{j=1}^{K-1} H^{(j)}(i,i), \tag{5.1}$$

where $i = 2^j + k + 1$ is unique for a given j and satifies the constrained Diophnatine equation: $2^j + 1 = k(2^{K-j} - 1) + p$, for some $k \in \{0, 1, ..., 2^j - 1\}$ and $p \in \{1, 2, ..., 2^{K-j}\}$.

Proof. Let H be a Haar wavelets matrix of size $n = 2^K (K > 1)$. We have

$$\text{tr}(H) = \sum_{i=1}^{2^K} H(i,i). \tag{5.2}$$

Clearly $t_1 \in C_1$ and $t_2 \in C_2$. Hence $H(1,1) = 1$ and $H(2,2) = 1$. Then above equation reduces to

$$\text{tr}(H) = 2 + \sum_{i=3}^{2^K} H(i,i). \tag{5.3}$$

For each $j \in \{1, 2, ..., K-1\}$, there are 2^j rows of H namely $2^j + 1, 2^j + 2, ..., 2^{j+1}$. Each row have 2^{K-j} number of nonzero columns and their pattern is explicitly given below.
Nonzero column indices in $(2^j + 1)^{th}$ row of H are: $1, 2, ..., 2^{K-j}$. i.e.,

$$H(2^j + 1, l) = \pm 1, l = 1, 2, ..., 2^{K-j}. \tag{5.4}$$

Nonzero columns in subsequent rows(up to $(2^{j+1})^{th}$) can be obtained by shifting nonzero columns in previous row by 2^{K-j} units. Among these 2^j rows, only one nonzero column lie on the diagonal of H, we denote this by $H^{(j)}(i,i)$. Hence (5.3) simplified to:

$$\text{tr}(H) = 2 + \sum_{j=1}^{K-1} H^{(j)}(i,i). \tag{5.5}$$

We complete the proof by exploring the relation between i and j in the above equation. For a fixed $j \in \{1, 2, ..., K-1\}$ and $i \in \{2^j + 1, 2^j + 2, ..., 2^{j+1}\}$. Recall $H(i,i) = \pm 1$ if and only if $t_i \in C_i$. i.e $t_i = t_{k(2^{K-j})+p}$ or $i = k(2^{K-j}) + p$, for some $p \in \{1, 2, ..., 2^{K-j}\}$. Hence, we have the following constrained Diophantine equation.

$$i^{th}\text{row} = i^{th}\text{nonzero column} \quad i.e., \; 2^j + k + 1 = k(2^{K-j}) + p, \tag{5.6}$$

where $k \in \{0, 1, ..., 2^j - 1\}$ and $p \in \{1, 2, ..., 2^{K-j}\}$. □

Remark 5.2. The trace of the matrix H of size 2^K can be computed by summing the $K+1$ nonzero values, among these, three nonzero values are: $H(1,1) = 1, H(2,2) = 1$ and $H(2^K, 2^K) = -1$. As a consequence of Theorem 5.1, an algorithm is proposed to compute the remaining the $K-2$ nonzero values ($K > 2$).

Algorithm to compute the trace of H

Input: $K \in \mathbb{N}(K > 2)$.

Output: $K-2$ nonzero values or $tr(H)$

Step 1: Choose $j = 1, 2, \ldots, K-2$. Denote $\Delta_j = 2^{K-j}$.

Solve the unique pair k and p (Theorem 5.1) using following constrained Diophantine equation:

$$2^j + 1 = k(\Delta_j - 1) + p;$$

for some k in $\{0, 1, \ldots, 2^{j-1}\}$ and p in $\{1, 2, \ldots, \Delta_j\}$.

Obtained k used for i, i.e., $i = 2^j + k + 1$. i.e., we picked the (i, i) entry in the band of 2^j rows such that $H(i, i) = \pm 1$. Obtained p used for deciding exactly 1 or -1 by ensuring that t_p in C_i^+ or C_i^- respectively.

$$H^{(j)}(i, i) = \begin{cases} 1, & if \quad p \leq \frac{\Delta_j}{2} \\ -1, & otherwise. \end{cases} \tag{5.7}$$

Step 2:

$$\mathrm{tr}(H) = 1 + \sum_{j=1}^{K-2} H^{(j)}(i, i). \tag{5.8}$$

Algorithm implemented using MATLAB programming for various sizes of H and the same is recorded in the Table 1.

K	tr(H)	K	tr(H)	K	tr(H)
2	1	12	11	22	21
3	0	13	8	23	10
4	3	14	9	24	21
5	0	15	8	25	20
6	5	16	15	26	21
7	2	17	8	27	18
8	5	18	17	28	27
9	4	19	10	29	16
10	9	20	15	30	29
11	2	21	16	31	22

Table 1: Trace of Haar matrices for different (2^K) sizes.

Goal of this article achieved by providing a counterexample to the result on the trace of Haar wavelets matrix existing in the literature and correspondingly amendment has done. During the journey of this goal, a fast algorithm developed to compute the trace of Haar matrix. Table 1. contains the amendement results on trace of Haar matrices with different sizes.

Dedication

This article is dedicated to Dr. V. A. Hiremath, Retired Professor of Mathematics, K. U. Dharwad for inspiring students to walk in the garden of counterexamples in Mathematics.

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References

[1] C.F. Chen and C.H. Hsiao, Haar wavelet method for solving lumped and distributed-parameter systems, *IEE Proc., Control Theory Appl.*, **144** (1997), 87-94.
[2] A. Haar, Zur theorie der orthogonalen Funktionsysteme, *Math. Ann.*, **69** (1910), 331–371.
[3] G. Hariharan and K. Kannan, An overview of Haar wavelet method for solving differential and integral equations, *World Applied Sciences Journal*, **23**(12) (2013), 01–14.
[4] U. Lepik and H. Hein, *Haar Wavelets With Applications*, Springer International publishing, 2014.
[5] P. Porwik and A. Lisowska, The HaarWavelet transform in digital image processing : Its status and achievements, *Machine GRAPHICS and VISION*, **13**(1/2) (2004), 79–98.
[6] S.C. Shiralashetti and S. Kumbinarasaiah, Some results on Haar wavelets matrix through linear algebra, *Wavel. Linear Algebra*, **4**(2) (2017), 49–59.